

Economics of Information

LECTURE 4

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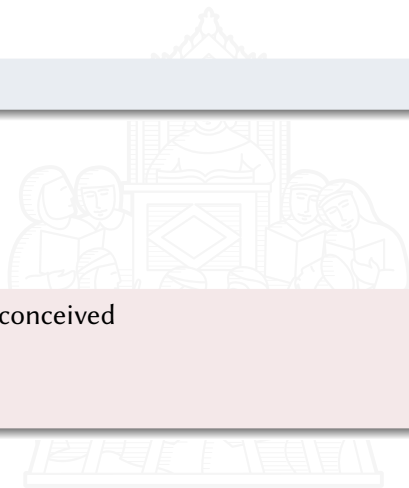
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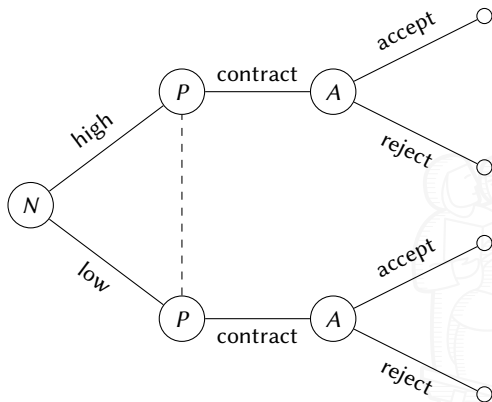
- game of **incomplete** information with **(un-)certainty**

- 1 Nature picks the agent type
- 2 the principal offers a contract
- 3 the agent accepts or rejects

- the agent has private information *before* the contract is conceived
- the principal might propose multiple contracts
- emphasis is on *which* contract the agent accepts



Adverse selection (cont'd)



Source: Rasmusen (2007, p. 183, fig. 7.1c)



Hidden actions vs. hidden knowledge

hidden actions

- agent's *effort* is non-contractible
- principal designs a contract that induces the agent to perform the desired *behaviour*

hidden knowledge

- agent's *ability* is non-contractible
- principal cannot induce a *characteristic*
- principal designs different contracts that are attractive to different types of agents in order to induce *self-selection*, **or not!**



Principal problem and equilibrium

the principal maximises her own utility knowing that

- the agent is free to reject the contract entirely
- the contract should induce the best possible *behaviour* or *selection*

hidden actions

- (participation)
- (incentive compatibility)

hidden knowledge

- (participation) $\times \#\{\text{agent types}\}$
- (self-selection) $\times \#\{\text{agent types}\}$

equilibrium types

if all types of agents choose the same strategy in all states, the equilibrium is **pooling**; otherwise it is **separating**

- sometimes it is too costly to induce self-selection



Production Game VI: adverse selection

players

principal: a manager

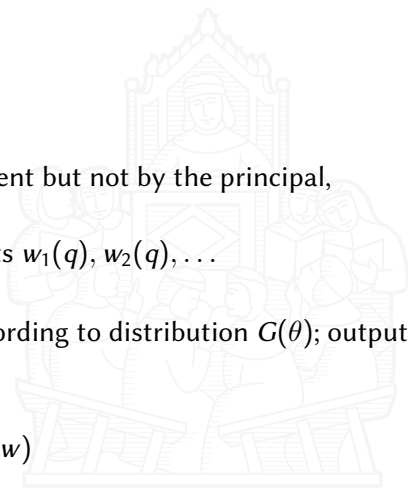
agent: a worker

order of play

- 0 Nature chooses the agent's ability a , observed by the agent but not by the principal, according to distribution $F(a)$
- 1 the principal offers the agent one or more wage contracts $w_1(q), w_2(q), \dots$
- 2 the agent accepts one contract or rejects them all
- 3 Nature chooses a value for the state of the world, θ , according to distribution $G(\theta)$; output is then $q = q(a, \theta)$

payoffs

- if agent accepts: $\pi_{agent} = U(w, a)$ $\pi_{principal} = V(q - w)$
- if agent rejects: $\pi_{agent} = \bar{U}(a)$ $\pi_{principal} = 0$



Production Game VI: adverse selection (cont'd)

- $F(a): \quad \Pr(a = 0) = 0.9, \quad \Pr(a = 10) = 0.1$
- $G(\theta) = \mathcal{U}\{0, 10\}$
- $q(a, \theta) = \min(a + \theta, 10)$
- $U(w) = w \quad \bar{U}_L = 3 \quad \bar{U}_H = 2 \quad V = q - w$



Production Game VI: adverse selection (cont'd)

- output can be either 0 or 10

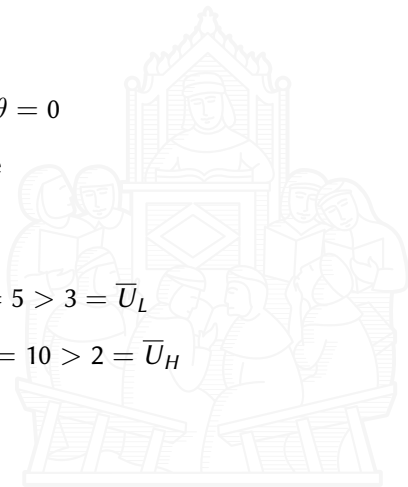
$$q(a, \theta) = \begin{cases} 0 & \text{if } a = 0 \wedge \theta = 0 \\ 10 & \text{otherwise} \end{cases}$$

- expected output of workers

$$E[q(a = 0, \theta)] = 0.5 \cdot 0 + 0.5 \cdot 10 = 5 > 3 = \bar{U}_L$$

$$E[q(a = 10, \theta)] = 0.5 \cdot 10 + 0.5 \cdot 10 = 10 > 2 = \bar{U}_H$$

- thus the principal *might* want to hire both types
- **N.B.** output and utility are comparable!



Production Game VI: adverse selection (cont'd)

- contracts are of the type

$$W_1 = \{w_1(q=0), w_1(q=10)\} \quad W_2 = \{w_2(q=0), w_2(q=10)\}$$

- W_1 targets a low-ability worker; W_2 targets a high-ability worker
- *participation* constraints: let $\pi_i(W_j)$ denote expected payoff for type i from contract j

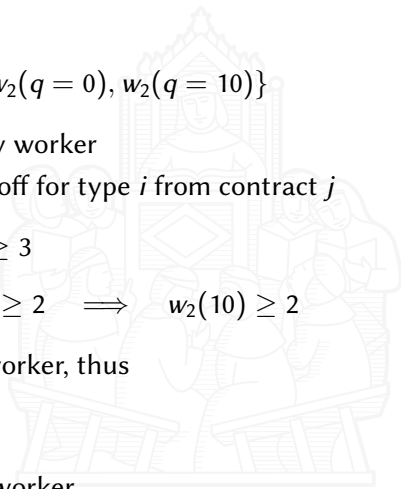
$$\pi_L(W_1) \geq \bar{U}_L \implies 0.5w_1(0) + 0.5w_1(10) \geq 3$$

$$\pi_H(W_2) \geq \bar{U}_H \implies 0.5w_2(0) + 0.5w_2(10) \geq 2 \implies w_2(10) \geq 2$$

- the low output wage doesn't matter to the high-ability worker, thus

$$w_2(q=0) = 0$$

- both low- and high-output wages matter for low-ability worker



Production Game VI: adverse selection (cont'd)

- first guess: the principal wants W_1 to be riskless to make sure low-ability worker accepts

$$w_1(q = 0) = 3 \qquad w_1(q = 10) = 3$$

- **Q:** how much should $w_2(q = 10)$ be? Can it be $w_2(q = 10) = 2$?
- *self-selection* constraints

$$\pi_L(W_1) \geq \pi_L(W_2) \implies 0.5w_1(0) + 0.5w_1(10) \geq 0.5w_2(0) + 0.5w_2(10)$$

$$\pi_H(W_2) \geq \pi_H(W_1) \implies w_2(10) \geq w_1(10)$$

- the high-ability worker has always the option of choosing W_1
- if $w_2(10) = 2$ then $\pi_H(W_2) \not\geq \pi_H(W_1)$; therefore $w_2(10) \geq 3$
- participation constraint is binding for bad type and not binding for good type



Production Game VI: adverse selection (cont'd)

- our best guess so far

$$W_1 = \{w_1(q=0) = 3, w_1(q=10) = 3\}$$

$$W_2 = \{w_2(q=0) = 0, w_2(q=10) = 3\}$$

- W_1 and W_2 satisfy the self-selection constraints

$$\pi_L(W_1) = 0.5 \cdot 3 + 0.5 \cdot 3 = 3 \geq 0.5 \cdot 0 + 0.5 \cdot 3 = 1.5 = \pi_L(W_2)$$

$$\pi_H(W_2) = 0.5 \cdot 3 + 0.5 \cdot 3 = 3 \geq 0.5 \cdot 3 + 0.5 \cdot 3 = 3 = \pi_H(W_1)$$

- self-selection constraint is binding for the good type and not binding for the bad type
- $w_2(q=10) = 3$ is a good choice because it ensures $W_2 \succsim_H W_1$ without overdoing (i.e. minimising cost for the principal)



Production Game VI: adverse selection (cont'd)

- expected profit for the principal

$$E[V] = 0.9 \cdot \underbrace{[0.5 \cdot \overbrace{(0 - 3)}^{q-w} + 0.5 \cdot \overbrace{(10 - 3)}^{q-w}]}_{\text{agent is low type}} + 0.1 \cdot \underbrace{(10 - 3)}_{\substack{\text{agent is} \\ \text{high type}}} = 2.5$$

- **Q:** what if W_1 were not riskless?
- all we need is that $\pi_L(W_1) \geq 3$
- if $W_1 = \{2, 4\}$ then self-selection constraint implies $W_2 = \{0, 4\}$

$$E[V] = 0.9 [0.5 \cdot (0 - 2) + 0.5 \cdot (10 - 4)] + 0.1 (10 - 4) = 2.4$$

- if $W_1 = \{4, 2\}$ then self-selection constraint implies $W_2 = \{0, 2\}$

$$E[V] = 0.9 [0.5 \cdot (0 - 4) + 0.5 \cdot (10 - 2)] + 0.1 (10 - 2) = 2.6$$

- **BOOK IS WRONG!**



Production Game VI: adverse selection (cont'd)

- both contracts satisfy participation constraint

$$\pi_L(W_1) \geq \bar{U}_L \implies 0.5 \cdot 4 + 0.5 \cdot 2 = 3 \geq 3$$

$$\pi_H(W_2) \geq \bar{U}_H \implies 2 \geq 2$$

- contrary to the previous case, the participation constraint is binding for both types
- both contracts satisfy self-selection constraint

$$\pi_L(W_1) \geq \pi_L(W_2) \implies 0.5 \cdot 4 + 0.5 \cdot 2 = 3 \geq 0.5 \cdot 0 + 0.5 \cdot 2 = 1$$

$$\pi_H(W_2) \geq \pi_H(W_1) \implies 2 \geq 2$$

- as before, self-selection constraint is binding for the good type
- it appears counter-intuitive to pay a higher wage when output is lower !
- but the additional loss in case agent type is L and state of the world is $\theta = 0$ is more than compensated by the additional gain in case either the agent type is H or $\theta = 10$



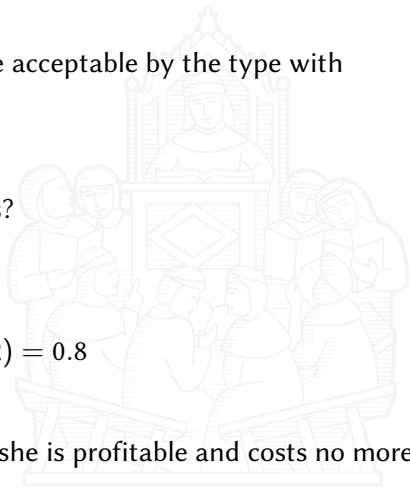
Production Game VI: adverse selection (cont'd)

- **Q:** would the principal prefer pooling?
- only participation constraint matters: contract should be acceptable by the type with highest reservation utility: $W = \{4, 2\}$ (or $W = \{3, 3\}$)
- but profit is the same as before

- **Q:** would the principal prefer to give up one of the types?
- suppose there is no type L
- then optimal contract is $W = \{0, 2\}$

$$E[V] = 0.9 \cdot 0 + 0.1 \cdot (10 - 2) = 0.8$$

- profit is lower than with both types
- principal doesn't want to give up H type either because she is profitable and costs no more than L type



The (basic) Lemons model

the principal negotiates the purchase of a car from the agent whose quality is non-contractible, despite lack of uncertainty

players

principal: a buyer

agent: a seller

order of play

- 0 Nature chooses seller's car quality $\theta \sim F(\theta)$
- 1 the buyer offers a price P
- 2 the seller accepts or rejects

payoffs

- if seller rejects: $\pi_{\text{buyer}} = \pi_{\text{seller}} = 0$
- if seller accepts: $\pi_{\text{buyer}} = V(\theta) - P, \quad \pi_{\text{seller}} = P - U(\theta)$



Lemons I: identical tastes, 2-types

- quality $\theta \sim \mathcal{U}\{2000, 6000\}$
- players value quality at one dollar per unit

$$V(\theta) = \theta$$

$$\pi_{\text{buyer}} = \theta - P$$

$$U(\theta) = \theta$$

$$\pi_{\text{seller}} = P - \theta$$

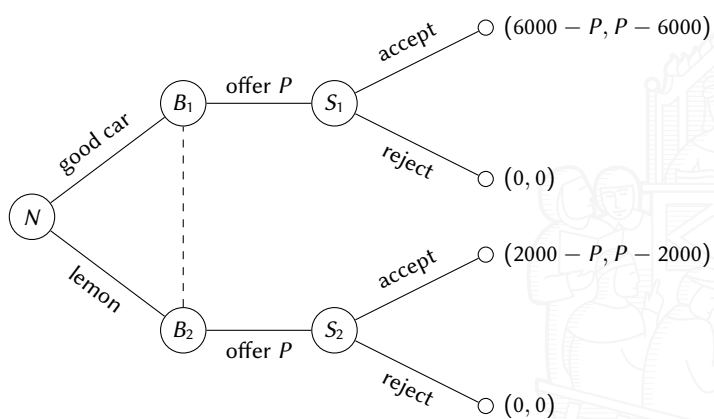
- quality is non-contractible, thus contract cannot be conditional
- the buyer cannot enforce a contract based on her discovery once the purchase is finalised

first guess

$$P = \text{average quality} = E[\theta] = 4000$$



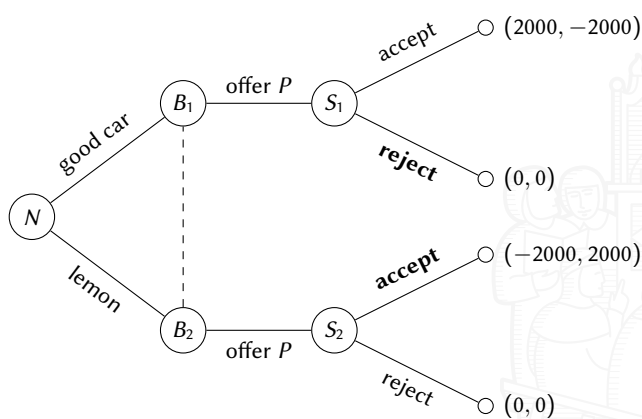
Lemons I: extensive form



Source: Rasmusen (2007, p. 250, fig. 9.1)



Lemons I: extensive form (cont'd)



Source: Rasmusen (2007, p. 250, fig. 9.1) with $P = 4000$



Lemons I: equilibrium

- if $P = 4000$ only sellers of lemons accept the contract
 - but the buyer is willing to pay up to $P = 2000$ for a lemon
 - if $P = 2000$
 - sellers of lemons are indifferent between accepting or rejecting P
 - buyers are indifferent between owning a car (lemon) or not
- the very fact that the car is for sale demonstrates its low quality
- in equilibrium only *half* of cars are traded, *all* of them lemons



Lemons II: identical tastes, continuum of types

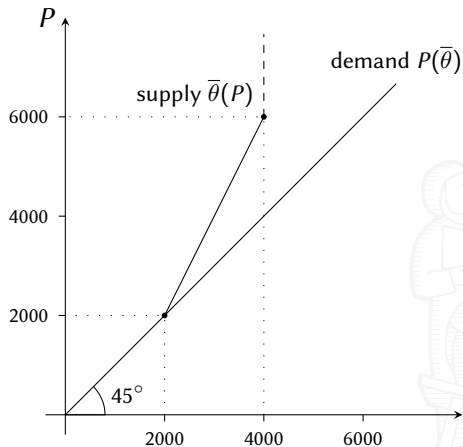
Q: was the outcome of **Lemons I** an artefact due to the 2-type assumption?

Lemons II

- *continuum of quality:* $\theta \sim \mathcal{U}(2000, 6000)$, $E[\theta] = 4000$
- as in **Lemons I**, if $P = 4000$, the seller is willing to sell only if $\theta \leq 4000$
 - sellers of cars with $\theta \in (4000, 6000]$ pull out of the market
 - average quality of car on sale is $E[\mathcal{U}(2000, 4000)] = 3000$
- P must drop to 3000
 - sellers of cars with $\theta \in (3000, 4000]$ pull out of the market
 - average quality of car on sale is $E[\mathcal{U}(2000, 3000)] = 2500$
- [...]
- P must drop to 2000
 - sellers of cars with $\theta > 2000$ pull out of the market
 - remaining # of cars with $\theta = 2000$ in the market is infinitesimal
- **the market has completely collapsed !**



Lemons II: equilibrium



Source: Rasmusen (2007, p. 251, fig. 9.2)

